Available online at www.sciencedirect.com
SCIENCE DIRECT®
PERGAMON
International Journal of Heat and Mass Transfer 46 (2003) 1251–1264
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Nonlinear coupling between thermal mass and natural ventilation in buildings

Jimmy Yam [a], Yuguo Li [a,*], Zuohuan Zheng [b]

[a] Department of Mechanical Engineering, The University of Hong Kong, Hong Kong SAR, China
[b] Institute of Applied Mathematics, Academy of Mathematics and Systems Sciences, Chinese Academy of Sciences, Beijing 100080, China

Received 2 July 2002

Abstract

An ideal naturally ventilated building model that allows a theoretical study of the effect of thermal mass associating with the non-linear coupling between the airflow rate and the indoor air temperature is proposed. When the ventilation rate is constant, both the phase shift and fluctuation of the indoor temperature are determined by the time constant of the system and the dimensionless convective heat transfer number. When the ventilation rate is a function of indoor and outdoor air temperature difference, the thermal mass number and the convective heat transfer air change parameter are suggested. The new thermal mass number measures the capacity of heat storage, rather than the amount of thermal mass. The analyses and numerical results show that the non-linearity of the system does neither change the periodic behaviour of the system, nor the behaviour of phase shift of the indoor air temperature when a periodic outdoor air temperature profile is considered. The maximum indoor air temperature phase shift induced by the direct outdoor air supply without control is 6 h.

Keywords: Thermal mass; Natural ventilation; Periodic heat flow; Night ventilation; Thermal coupling; Buildings; Passive design

1. Introduction

Thermal mass or its thermal storage effect can be used to reduce energy consumption of mechanical cooling and heating systems in buildings. When a building is naturally ventilated, thermal mass can be used to regulate indoor air temperatures. Passive design with the use of thermal storage can become very effective when there is a significant diurnal variation of ambient temperature and/or diurnal variation in solar radiation intensity. The working principle is very simple that thermal mass stores heat in both the building envelope materials and the interior mass such as partitions, ceiling and floor during a warm period on a summer day and releases it at a later time in the day. The peak cooling loads can be reduced thereby similarly, the stored heat during high solar gains can be released into the building in the late afternoon, which can satisfy partly the heating needs during cold period. Two engineering questions are to be interested as how much thermal mass should be used in a particular design and what are the quantitative impacts of thermal mass on cooling/heating loads and indoor air temperature. The main focus of this paper is to study the non-linear coupling between ventilation and *internal* thermal mass in naturally ventilated buildings. Internal thermal mass such as furniture and purposed-built internal concrete partitions does not expose to ambient temperature directly, while the external thermal mass such as walls and roofs expose directly to ambient temperature variation.

Balaras [1] reviewed a large number of the previous studies on thermal mass effects in buildings. Different simplified models for taking into account the building's thermal mass into cooling load analyses were reviewed, and different parameters were found for describing the

* Corresponding author. Tel.: +852-2859-2625; fax: +852-2858-5415.
E-mail address: liyg@hku.hk (Y. Li).

0017-9310/03/$ - see front matter
PII: S0017-9310(02)00379-4

Nomenclature

A_b	area of the bottom opening 'b', m^2
A_M	effective heat transfer area of the thermal mass, m^2
A_t	area of the top opening 't', m^2
A^*	effective opening area of a building, m^2
B	buoyancy flux, m^4/s^3
C_d	discharge coefficient
C_M	heat capacity of the thermal mass, J/kg °C
C_p	heat capacity of air, J/kg °C
E	effective total heat power, W
g	acceleration of gravity, m/s^2
h	height between two vertical openings 't' and 'b', m
h_M	convective heat transfer coefficient at surface of thermal mass materials, $W/m^2\,°C$
M	mass of thermal mass, kg
q	ventilation flow rate, m^3/s
t	time, h
T_E	air temperature rise due to steady state heat source, K
T_i	indoor air temperature, K
T_M	thermal mass temperature, K
T_o	outdoor air temperature, K
$\widetilde{T}_o$	mean outdoor air temperature, K
$\Delta\widetilde{T}_o$	amplitude of fluctuation of outdoor air temperature, K

Greek symbols

α	buoyancy air change parameter, m^3/s
β	phase shift
φ	thermal mass number
λ	convective heat transfer number
θ	outdoor temperature fluctuation air change parameter, m^3/s
θ_E	heat source induced air temperature parameter, K
θ_H	convective heat transfer air change parameter, m^3/s
ρ	air density, kg/m^3
τ	time constant, h
ω	frequency of outdoor temperature variation, h^{-1}

Subscripts

b	bottom opening
i	indoor
M	thermal mass
o	outdoor
t	top opening

thermal mass effects [2–4]. Mathews et al. [3] provided a procedure for estimating the effective heat storage capacity in a building. More detailed treatment of thermal mass effect was also done through analytical and numerical solutions of transient heat transfer through external thermal mass such as walls. Classical texts in analytical heat conduction such as Carslaw and Jaeger [5] have found their use in buildings through the well-known work such as Danter [6].

Thermal mass is effective for dampening the wide range temperature fluctuation from the outdoor and maintaining the indoor air temperature within a comfortable range [7–9]. The heat storage in thermal mass works together with other heat transfer processes in buildings. Heat is also gained or lost in a building through heat conduction through the building envelope, thermal radiation through windows, ventilation through openings and/or infiltration/exfiltration through leakages. The so-called night ventilation technique [10,11] is based on the principle of ventilating the building at night to cool down its walls, floor and ceiling as the thermal mass, and absorbing the heat during the following daytime. Night ventilation can be either natural or mechanical. When it is mechanical, the analyses are much simpler than when it is naturally ventilated. Kammerud et al. [12] presented a study of the effect of ventilation cooling for a group of residential buildings. Their analyses assumed a fan-forced ventilation rather than natural ventilation.

There are at least two associated difficult issues when dealing with natural ventilation. Firstly, the ventilation flow rate is not a constant, as the ventilation opening such as window opening is affected by human behavior. Secondly, the natural ventilation flow rates also depend on changing wind and thermal forces. For stack-driven natural ventilation, the ventilation flow rate depends on the temperature difference between indoor and outdoor air, while at the meantime the indoor air temperature is also a function of the ventilation flow rate. Ventilation flow rate and indoor air temperature are coupled in a nonlinear manner. Almost all existing studies have treated the problem as a linear system. Based on a linear approach, Mathews [13] developed a simplified electric analogue method, and it is obvious that the indoor air temperature changes periodically if the outdoor air temperature varies periodically. However, this is not straightforward when the governing equation is nonlinear. The questions include whether the indoor air temperature also varies periodically, and if it changes periodically, then, what is the phase difference between the indoor and outdoor air temperatures and what are the engineering parameters affecting the phase shift.

Van der Maas and Roulet [10] developed a simple dynamic model that couples airflow, heat transfer and a thermal model for the wall, numerical method was used to solve the coupled equations. With some modifications, the present analyses are also applicable to situations when phase change materials are used for thermal storage.

2. A simple building model with constant ventilation flow rate

We first consider an ideal building model with a constant ventilation flow rate (see Fig. 1). The building is ventilated mechanically. The air temperature distribution in the building is uniform, implying that the airflow is fully mixed. The building envelope is perfectly insulated and due to the uniformity of indoor air temperature, the thermal radiation between the room surfaces do not exist. All heat gain and heat generation in the building can be lumped into one heat source term, E.

The temperature distribution in the thermal mass materials is also assumed to be uniform. This means that the thermal diffusion process is much faster than the convective heat transfer at thermal mass surface. Two situations are considered here as shown in Fig. 1.

1. The thermal mass materials are in equilibrium with the indoor air. This means that the thermal mass temperature is always the same as the indoor air temperature (Fig. 1a). This assumption allows a simple governing equation to be derived.
2. The thermal mass materials are not in equilibrium with the indoor air (Fig. 1b). Then, the convective heat transfer process between the thermal mass and indoor air should be considered.

2.1. The thermal mass is in equilibrium with the room air

The heat balance equation for the building becomes

$$\omega M C_{\mathrm{M}} \frac{\partial T_{\mathrm{i}}}{\partial(\omega t)} + \rho C_p q (T_{\mathrm{i}} - T_{\mathrm{o}}) = E \tag{1}$$

The ventilation flow rate q is always positive. In this paper, we do not study the effect of heat transfer through the building envelope, and hence, the effect of solar radiation is not an issue here. We assume that the outdoor temperature can be expressed by Fourier analysis as the sum of sinusoidal components of periods 24, 12, 8, 6, etc., hours. Similar to other studies, we only consider the main sinusoidal component of period 24 h.

$$T_{\mathrm{o}} = \widetilde{T}_{\mathrm{o}} + \Delta\widetilde{T}_{\mathrm{o}} \sin(\omega t) \tag{2}$$

where $\Delta\widetilde{T}_{\mathrm{o}}$ and $\widetilde{T}_{\mathrm{o}}$ are independent of time and $\Delta\widetilde{T}_{\mathrm{o}} \geqslant 0$; ω is the frequency of the outdoor temperature fluctuation with a value of $2\pi/24\ \mathrm{h}^{-1}$.

Substituting Eq. (2) into (1) and after some rearrangements, we have

$$\omega\tau \frac{\partial T_{\mathrm{i}}}{\partial(\omega t)} + T_{\mathrm{i}} = T_E + \widetilde{T}_{\mathrm{o}} + \Delta\widetilde{T}_{\mathrm{o}} \sin(\omega t) \tag{3}$$

where $\tau = MC_{\mathrm{M}}/\rho C_p q$ and $T_E = E/\rho C_p q$. The system as described by Eq. (3) is similar to the classical lumped parameter model of a solid body with convective heat transfer at its surface [14]. The general solution of the differential equation (3) can be written as

$$T_{\mathrm{i}}(\omega t) = \widetilde{T}_{\mathrm{o}} + T_E + \frac{\Delta\widetilde{T}_{\mathrm{o}}}{1+\omega^2\tau^2} \sin\omega t - \frac{\omega\tau\Delta\widetilde{T}_{\mathrm{o}}}{1+\omega^2\tau^2} \times \cos\omega t + C\mathrm{e}^{-(1/\tau)t} \tag{4}$$

where C is a constant, determined by the initial conditions. τ is the time constant of the system. The solution is the sum of two parts. The first part is a periodic solution with a period of 2π. The second part contains the initial condition, which decays to zero as time increases. In terms of dynamical systems, the solution (4) is said to be a global attractor. The solution (4) oscillates non-periodically, as $C \neq 0$, approaching to the following periodic solution.

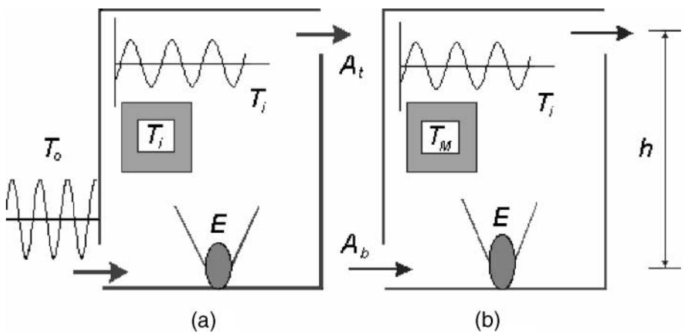

Fig. 1. A simple two-opening one-zone building model with periodic outdoor air temperature variation. The shaded area represents the thermal mass. (a) The thermal mass is in equilibrium with the room air. (b) The thermal mass is not in equilibrium with the room air.

$$T_P(\omega t) = \widetilde{T}_o + T_E + \frac{\Delta\widetilde{T}_o}{\sqrt{1+\omega^2\tau^2}}\sin(\omega t - \beta) \tag{5}$$

where the phase shift $\beta = \tan^{-1}(\omega\tau)$, with $\sin\beta = \omega\tau/\sqrt{1+\omega^2\tau^2}$ and $\cos\beta = 1/\sqrt{1+\omega^2\tau^2}$.

After sufficient long time, the indoor air temperature becomes periodic, which has three components: the first is the mean outdoor air temperature, the second is the steady state air temperature rise due to the steady heat source, the third is the fluctuating component. If the outdoor air temperature fluctuation is zero, then the third part is zero. The magnitude of the indoor air temperature fluctuation is always smaller than that of the outdoor air temperature fluctuation with a coefficient of $1/\sqrt{1+\omega^2\tau^2}$. The amplitude reduction is a function of both the frequency of outdoor temperature fluctuation $\Delta\widetilde{T}_o$ and the time constant τ of the system (see Fig. 3). For the 24-h frequency alone considered here, it can be found when the time constant τ of the system is large, the fluctuation of indoor air temperature becomes small. We sketch a typical indoor air temperature profile in Fig. 2.

The phase shift β of the indoor air temperature is also a function of both the frequency of outdoor temperature fluctuation and the time constant. It has a value between 0 and $\pi/2$ (i.e. 0 and 6 h, see Fig. 4). When the time constant τ approaches zero, the indoor air temperature follows right along in phase with the outdoor temperature. As time constant approaches infinity, the indoor air temperature is out of phase by $\pi/2$ (6 h). The phase shift β can be expressed as

$$\beta = \frac{24}{2\pi}\tan^{-1}\left(\frac{\omega M C_M}{\rho C_p q}\right) = \frac{12}{\pi}\tan^{-1}\left(\frac{\omega M C_M}{\rho C_p q}\right) \quad \text{(hours)} \tag{6}$$

It can be seen that to achieve the same phase shift, the amount of thermal mass should be proportional to the amount of ventilation flow rate. A higher ventilation rate in a building requires a larger amount of thermal mass.

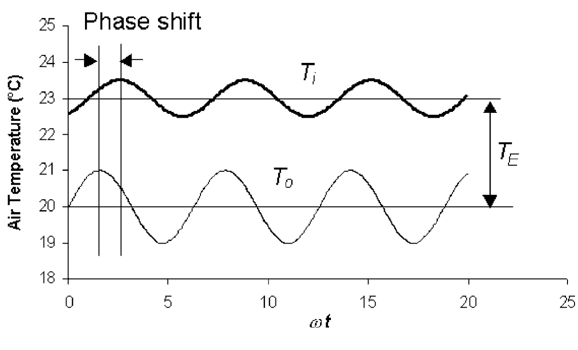

Fig. 2. A sketch of the periodic indoor and outdoor air temperature profiles in a simple building when the ventilation flow rate is constant.

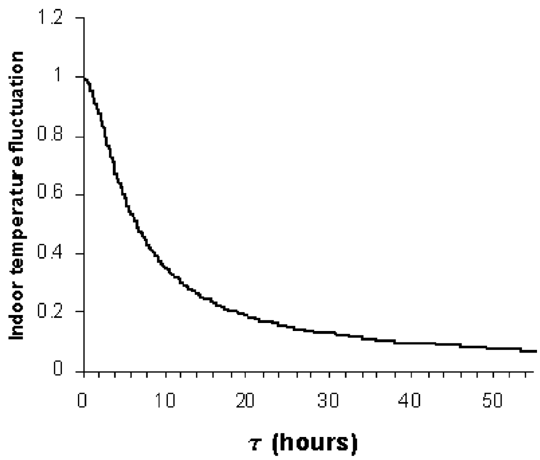

Fig. 3. The non-dimensional indoor air temperature fluctuation $\Delta\widetilde{T}_i$ (normalized by the outdoor air temperature fluctuation $\Delta\widetilde{T}_o$) as a function of the time constant.

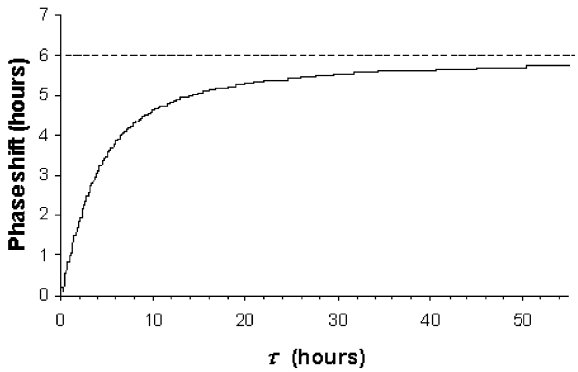

Fig. 4. The phase shift of the indoor air temperature as a function of the time constant.

2.2. *The thermal mass is not in equilibrium with the room air (Fig. 1b)*

A more realistic situation is the thermal mass materials being not in equilibrium with the indoor air. We assume that the thermal mass has a uniform temperature distribution.

There are two basic heat balance equations, one for the room air and one for the thermal mass.

$$\rho C_p q(T_o - T_i) + h_M A_M (T_M - T_i) + E = 0 \tag{7}$$

$$MC_{\rm M}\frac{\partial T_{\rm M}}{\partial t}+h_{\rm M}A_{\rm M}(T_{\rm M}-T_{\rm i})=0 \tag{8}$$

From Eq. (7), we obtain

$$T_{\rm M}=\left(1+\frac{\rho C_p q}{h_{\rm M}A_{\rm M}}\right)T_{\rm i}-\frac{\rho C_p q}{h_{\rm M}A_{\rm M}}T_{\rm o}-\frac{E}{h_{\rm M}A_{\rm M}} \tag{9}$$

Substituting Eq. (9) into (8), after some manipulation, we obtain

$$\omega\tau\frac{\partial T_{\rm i}}{\partial(\omega t)}+\frac{\lambda}{1+\lambda}T_{\rm i}=\frac{\lambda}{1+\lambda}\left(\widetilde{T}_{\rm o}+T_E\right)+\frac{\lambda}{1+\lambda}\Delta\widetilde{T}_{\rm o}\times\left[\sin(\omega t)+\left(\frac{\omega\tau}{\lambda}\right)\cos(\omega t)\right] \tag{10}$$

where $\lambda=h_{\rm M}A_{\rm M}/\rho C_p q$, $\tau=MC_{\rm M}/\rho C_p q$ and $T_E=E/\rho C_p q$. A new convective heat transfer number λ is introduced to measure relative strength of convective heat transfer at the thermal mass surface. If λ becomes infinity, i.e. the convective heat transfer is infinitely effective, Eq. (10) can be shown to be identical to Eq. (3), which is the governing equation for the situation when the thermal mass is in equilibrium with the indoor air.

The general solution for Eq. (10) is

$$T_{\rm i}(\omega t)=\widetilde{T}_{\rm o}+T_E+\sqrt{\frac{\lambda^2+\omega^2\tau^2}{\lambda^2+\omega^2\tau^2(1+\lambda)^2}}\Delta\widetilde{T}_{\rm o}\times\sin(\omega t-\beta)+C{\rm e}^{-(\lambda/\omega\tau(1+\lambda))\omega t} \tag{11}$$

where C is an integrating constant and $\beta=\tan^{-1}[\lambda^2\omega\tau/(\lambda^2+\omega^2\tau^2(1+\lambda))]$. As the convective heat transfer number λ becomes infinity, $\beta=\tan^{-1}(\omega\tau)$, which agrees with the analysis in Section 2.1.

After sufficient long time, the solution approaches to a periodic one as

$$T_{\rm i}(\omega t)=\widetilde{T}_{\rm o}+T_E+\sqrt{\frac{\lambda^2+\omega^2\tau^2}{\lambda^2+\omega^2\tau^2(1+\lambda)^2}}\Delta\widetilde{T}_{\rm o}\sin(\omega t-\beta) \tag{12}$$

The first term is the mean outdoor temperature and the second term is the steady-state air temperature rise due to steady heat source. The mean indoor air temperature $(\widetilde{T}_{\rm o}+T_E)$ is not a function of the convective heat transfer number and the time constant of the system. The third term is the periodic fluctuating component with its amplitude depending on the outdoor temperature fluctuation $\Delta\widetilde{T}_{\rm o}$, the time constant τ and the convective heat transfer number λ. β is again the phase lag of the indoor air temperature with respect to the outdoor temperature.

Analytical solution (12) is plotted in Figs. 5 and 6 for the phase shift and the fluctuation amplitude of the indoor air temperature respectively. The solution behaviors at extreme conditions can be easily explained as follows. If $\lambda\to\infty$, both the phase shift and the fluctuating component of indoor air temperature are identical to those obtained in Section 2.1. This is due to that the heat transfer resistance between the thermal mass and the indoor air becomes negligible. Thus the thermal mass temperature is in equilibrium with the indoor air temperature. As the thermal mass increases, both the time constant and the convective heat transfer number increase. It can be shown from the analytical solution (12) that as both the time constant and convective heat transfer number approach to infinity, the indoor air temperature fluctuation approaches to zero.

If the convective heat transfer at the thermal mass surface is absent, then $\lambda=0$, the phase shift of the indoor air temperature is zero and its fluctuation is exactly the same as that of the outdoor air temperature, no matter how large is the thermal mass value. This is

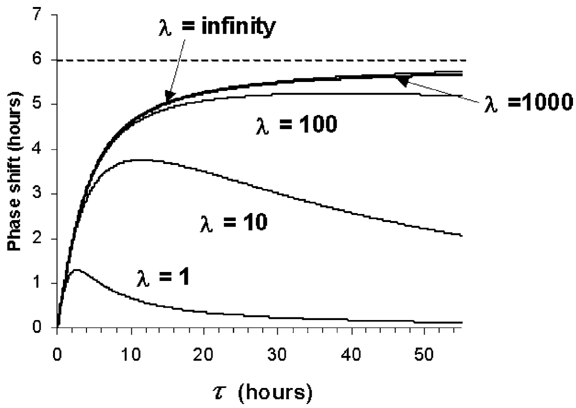

Fig. 5. The phase shift of the indoor air temperature as a function of the time constant and the convective heat transfer number.

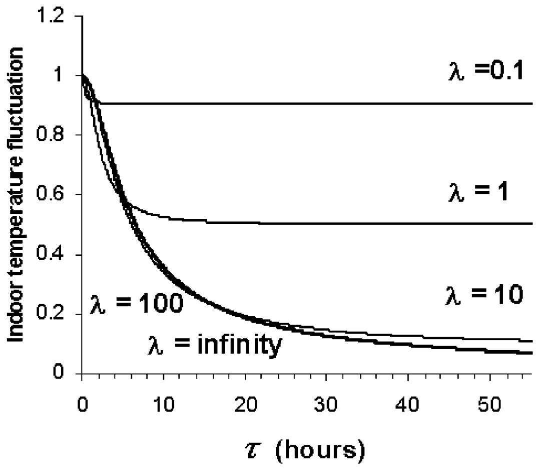

Fig. 6. The non-dimensional indoor air temperature fluctuation $\Delta\widetilde{T}_{\rm i}$ (normalized by the outdoor air temperature fluctuation $\Delta\widetilde{T}_{\rm o}$) as a function of the time constant and the convective heat transfer number.

simply because there is no longer any thermal interaction between the thermal mass and the indoor air.

It is not difficult to understand, that when λ is small (between 0.1 to 10, which are typical practical values), the phase shifts are much smaller than those with very large convective heat transfer numbers. However, it is not obvious that for a fixed value of λ, the phase shift first increases exponentially as the time constant increases, then drops as the time constant further increases, approaching zero as the time constant approaches infinity. It can be seen from Fig. 6 that, for λ ranging from 0.1 to 1, the fluctuation amplitude of the indoor air temperature, normalized by the outdoor fluctuation amplitude, becomes constant as the time constant becomes very large. This suggests that the convective heat transfer between the mass and nearby air is an important aspect in thermal mass design, which is well known by engineers.

3. A simple building model with stack-driven ventilation

For the simple building model shown in Fig. 1, the basic assumptions are the same as in Section 2, except that the stack-driven ventilation flow rate is not a constant. Two openings at different vertical levels on opposite walls, are relatively small, and the areas of the top and bottom opening are A_t and A_b respectively. With stack driven ventilation, it is known that indoor airflow is thermally stratified in some circumstances. However, the fully mixed assumption is used here because it leads to relatively simple equations, which nonetheless display interesting behaviour and because this assumption is used in the simpler treatments of natural ventilation of multi-zone buildings.

Let the ventilation flow rate, q, to be positive for upward ventilation flows and negative for downward flows. Noting that the indoor temperature can be either higher or lower than the outdoor temperature. According to Li and Delsante [15],

$$q = C_d A^* \mathrm{sgn}(T_i - T_o)\sqrt{\left|2gh\frac{T_i - T_o}{\widetilde{T}_o}\right|} \tag{13}$$

where C_d is the opening discharge coefficient assumed to be the same for both openings, $A^* = A_t A_b / \sqrt{A_t^2 + A_b^2}$ is the effective area, and h is the vertical distance between the top and bottom openings. The positive sign is used in Eq. (13) if indoor air temperature is higher than outdoor temperature, while the negative sign is used if the indoor air is cooler. The volumetric expansion coefficient is assumed to be constant. This ventilation flow rate equation can be rewritten as

$$q|q| = (C_d A^*)^2 2gh \frac{(T_i - T_o)}{\widetilde{T}_o} \tag{14}$$

Again, two situations are also considered below.

3.1. The thermal mass is in equilibrium with the room air

The heat balance for the single-zone building gives

$$\omega M C_M \frac{\partial T_i}{\partial(\omega t)} + \rho C_p |q| (T_i - T_o) = E \tag{15}$$

Note that the ventilation flow rate can be either negative or positive.

Combining Eqs. (14) and (15), after some rearrangement, we obtain the following governing equation for the ventilation flow rate,

$$2\varphi\theta|q|\frac{\partial q}{\partial(\omega t)} = -q^3 + 2\alpha^3[1 - \varphi\cos(\omega t)] \tag{16}$$

where

$$\alpha = (C_d A^*)^{2/3}(Bh)^{1/3} \tag{17}$$

$$B = \frac{Eg}{\rho C_p \widetilde{T}_o} \tag{18}$$

$$\theta = \frac{E}{\rho C_p \Delta\widetilde{T}_o} \tag{19}$$

$$\varphi = \frac{M C_M \omega \Delta\widetilde{T}_o}{E} \tag{20}$$

The parameter α is referred as the buoyancy air change parameter in the literature and it characterizes the effect of the thermal buoyancy force [15]. B is the buoyancy flux. α is found to be useful for analyzing buoyancy-driven natural ventilation. Two new parameters φ and θ are emerged to characterize the effect of thermal mass and the outdoor temperature fluctuation. The thermal mass number φ is dimensionless, and is the ratio between the maximum heat stored $M C_M \omega \Delta\widetilde{T}_o$ by thermal mass and the total heat gain in the building. θ is the outdoor temperature fluctuation air change parameter, with the same dimension as the airflow rate. As the ventilation flow rate is not known, we cannot define a time constant as in the linear problem.

Similarly, we can obtain the governing equation for indoor air temperature T_i. From Eqs. (14) and (15)

$$\frac{\varphi}{\sqrt[3]{2}}\left(\frac{\theta}{\alpha}\right)\frac{\partial Y}{\partial(\omega t)} + Y|Y|^{1/2} = 1 - \varphi\cos(\omega t) \tag{21}$$

where $Y = (T_i - T_o)/\theta_E$ and θ_E is the heat source induced air temperature parameter, which has a unit of temperature (K) and is defined as follows,

$$\theta_E = \left[\frac{E^2 \widetilde{T}_o}{2gh(C_d A^*)^2 \rho^2 C_p^2}\right]^{1/3} \tag{22}$$

There are no known general solutions to the non-linear equations (16) and (21). Here we try to illustrate what happens to the indoor air temperature when there is a non-linear interaction between the ventilation flow rate and thermal mass.

We examine first the situation for φ approaching infinity. Eq. (16) becomes

$$2\theta|q|\frac{\partial q}{\partial(\omega t)} = -2\alpha^3\cos(\omega t) \tag{23}$$

The general solution of the differential equation (23) is

$$q|q| = -\frac{2\alpha^3}{\theta}\sin(\omega t) + C \tag{24}$$

where C is a constant. The phase difference between the ventilation flow rate q and the outdoor air temperature T_o is thus $\frac{24}{2\pi}\left(\frac{3\pi}{2}-\frac{\pi}{2}\right) = 12$ h. We are more interested in the maximum phase shift for the indoor air temperature. The solution for the indoor air temperature can be easily obtained by either using Eqs. (14) and (24) or by solving Eq. (21) directly, we have

$$T_i(\omega t) = \left[1 + \frac{C}{2gh(C_d A^*)^2}\right]\widetilde{T}_o \tag{25}$$

where C is a constant. This shows that as the amount of thermal mass approaches infinity, the indoor air temperature does not vary in time. The largest phase shift occurs when the thermal mass number φ is the largest, i.e. approaching infinity. The question is what is the phase shift for indoor air temperature as thermal mass approaches infinity. Obviously, the analytical solution (25) cannot give the answer, and more mathematical analyses as summarized in Appendix A are needed. It was obtained that the solution for the nonlinear governing equation (16) is a periodic oscillating solution with a 24-h period after sufficient time and the phase difference between the periodic oscillating solution of the airflow rate and the outdoor air temperature T_o are within 6–12 h. Similar analysis can be done for the governing equation of the indoor air temperature, Eq. (25), and the result is also periodic with a period of 24 h and the phase shift is between 0 and 6 h. Unfortunately, we cannot prove mathematically at the present that the periodic solution is unique, although all numerical solutions obtained so far are unique.

The governing equations (16) and (21) are solved by the fourth-order Runge–Kutta method. The ventilation flow and the steady-state periodic temperature profiles for a naturally ventilated building model with a very large thermal mass number are shown in Fig. 7, from which there are time lags for both the indoor air temperature (solid line) and ventilation flow (dash-dotted line) with respect to the outdoor temperature (dotted line). Also the fluctuation amplitude of the indoor air temperature is much smaller than that of the outdoor temperature.

Fig. 8 shows the variation of the phase shifts for both the flow rate and the indoor temperature compared to the outdoor air temperature. Fig. 8 was plotted using both the airflow rate equation (16) and the indoor air temperature equation (21), and the results were identical. The numerical results agree well with the mathematical analyses in Appendix A. The phase shift of the airflow rate is bounded by 6 and 12 h and the phase shift for the indoor air temperature ranges from 0 to 6 h. For both phase shifts, the rates of increase are initially very high and then gradually decrease until the phase shifts become constants. This indicates that the use of thermal mass should be optimized, as excessive thermal mass is

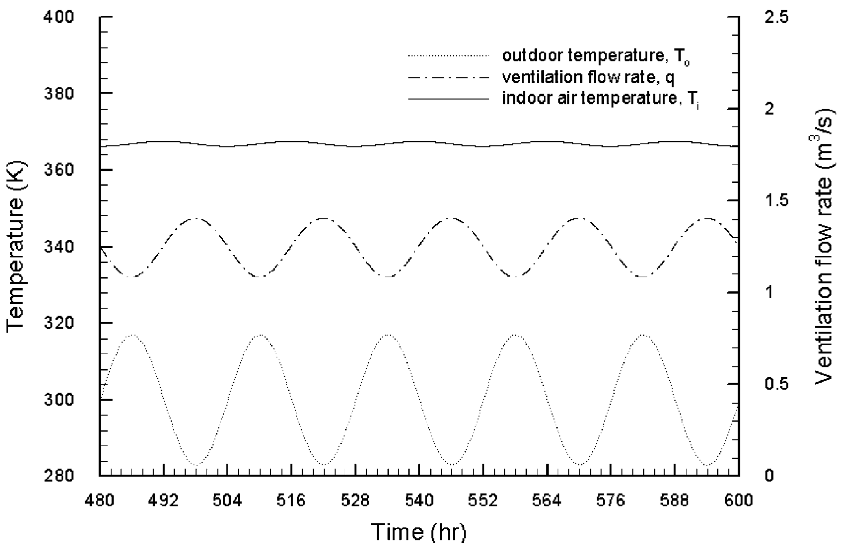

Fig. 7. A sketch of the ventilation flow and temperature profiles of a naturally ventilated building model with thermal mass effect.

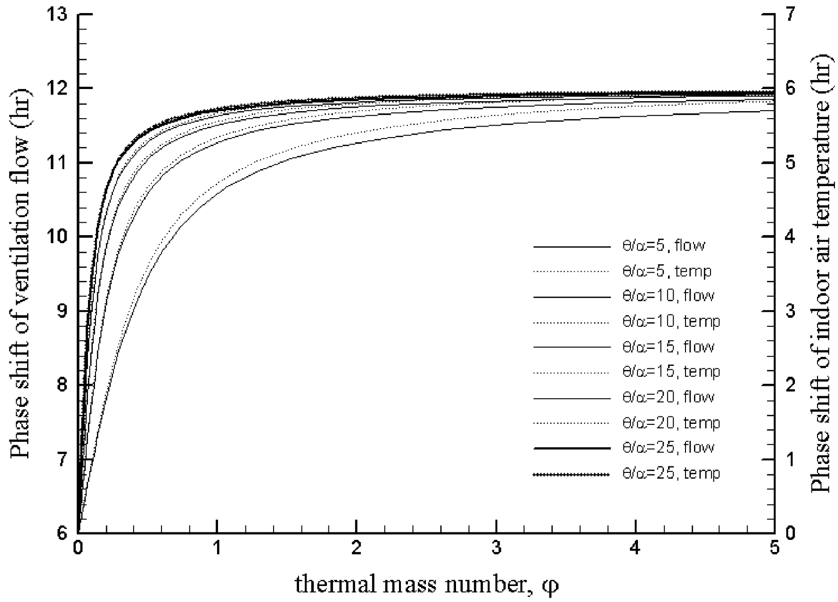

Fig. 8. Phase shifts for both the ventilation flow rate (solid lines) and the indoor air temperature (dotted lines) for the stack-ventilated model with $T_M = T_i$. Note two vertical axis are used.

not beneficial when the phase shift approaches to its maximum value. As shown in Fig. 8, the higher the value of θ/α, the greater is the phase shift for the same thermal mass number.

For the indoor temperature, the phase shift approaches to zero as φ approaches to zero (Fig. 8). This is obvious, but can also be shown by the analysis of the governing equation. Substituting $\varphi = 0$ into Eq. (16) gives

$$q = \sqrt[3]{2}\alpha = q_{ss}$$

where $q_{ss} = \sqrt[3]{2}\alpha$ is the steady-state flow rate for a single-zone building under stack-driven ventilation alone [15] and without thermal mass effect. The indoor air temperature, T_i, is obtained as

$$T_i = \left(\widetilde{T}_o + \frac{E}{\rho C_p q_{ss}}\right) + \Delta\widetilde{T}_o \sin(\omega t) \tag{26}$$

T_i is a sinusoidal function in phase with the outdoor temperature T_o.

Figs. 9 and 10 show respectively the variations of the predicted mean values and the fluctuation (amplitude) of the ventilation flow rate, normalized by the thermal air change parameter, α. When the effect of thermal mass is absent (i.e. $\varphi = 0$), the ventilation flow rate equals $\sqrt[3]{2}\alpha$ and no fluctuation exists. Examining the curve for $\theta/\alpha = 5$ in Fig. 9, it can be seen that as φ increases, i.e. the amount of relative heat storage increases, the mean ventilation flow rate first decreases exponentially and then approaches to a constant as the thermal mass number is greater than 2. This is due to the fact that the heat is stored in the thermal mass and the mean temperature difference between the indoor and outdoor is reduced. In the meantime, examining the thick solid curve ($\theta/\alpha = 5$) in Fig. 10, the fluctuation first increases exponentially as a result of increasing thermal mass, and then approaches to a constant.

Eq. (6) shows that for constant ventilation flow rate, the indoor air temperature fluctuation is a linear function of the outdoor air temperature fluctuation. It seems that, even when the ventilation flow rate is not constant, the indoor air temperature fluctuation also increases as the outdoor air temperature fluctuation increases (see Fig. 11). This is also reflected in the trend of ventilation flow rate fluctuation when the outdoor air temperature fluctuation decreases (Fig. 10). A decreasing outdoor air temperature fluctuation means an increasing outdoor air temperature fluctuation parameter θ. It is obvious that as the outdoor air temperature fluctuation reduces to zero, the fluctuation of the outdoor airflow rate also becomes zero.

Fig. 11 shows the changes of the ratio of indoor temperature fluctuation to the outdoor temperature fluctuation with respect to thermal mass number φ. As shown, the ratios decrease from 1 to 0 as φ increases from zero to infinity. This agrees well with the physical features of the system, the indoor air temperature profile follows in phase with the outdoor air temperature variation as the effect of thermal mass is absent, and the outdoor air temperature fluctuation will be dampened completely (as the effect of thermal mass is very large (approaching infinity)).

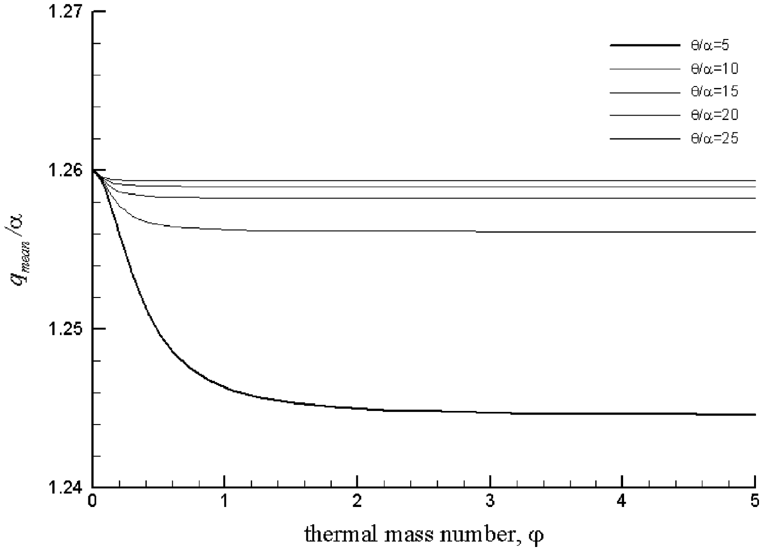

Fig. 9. Normalized mean ventilation rate as a function of the thermal mass number for the stack-ventilated model with $T_M = T_i$.

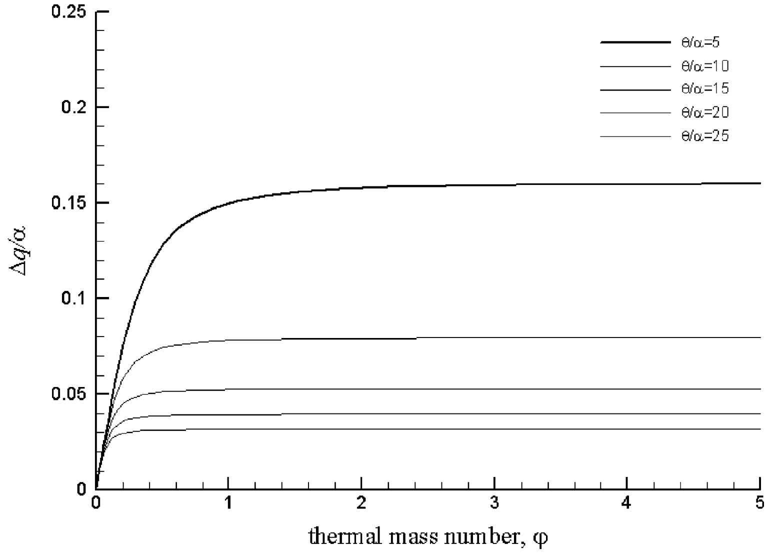

Fig. 10. Normalized ventilation rate fluctuation as a function of the thermal mass number for the stack-ventilated model with $T_M = T_i$.

3.2. The thermal mass is not in equilibrium with the room air

Again, there are two basic heat balance equations, one for the room air and another for the thermal mass, as

$$\rho C_p|q|(T_o - T_i) + h_M A_M (T_M - T_i) + E = 0 \qquad (27)$$

$$MC_M \frac{\partial T_M}{\partial t} + h_M A_M (T_M - T_i) = 0 \qquad (8)$$

Eq. (27) differs only from Eq. (7) that the flow rate, q, can be either positive or negative.

Using Eq. (27) and the flow rate equation (14), we obtain

$$T_M = \frac{E}{h_M A_M}\left[\left(\frac{T_i - T_o}{\theta_E}\right)\left|\frac{T_i - T_o}{\theta_E}\right|^{1/2} - 1\right] + T_i \qquad (28)$$

Substituting Eq. (28) into (8), after simplification, we have

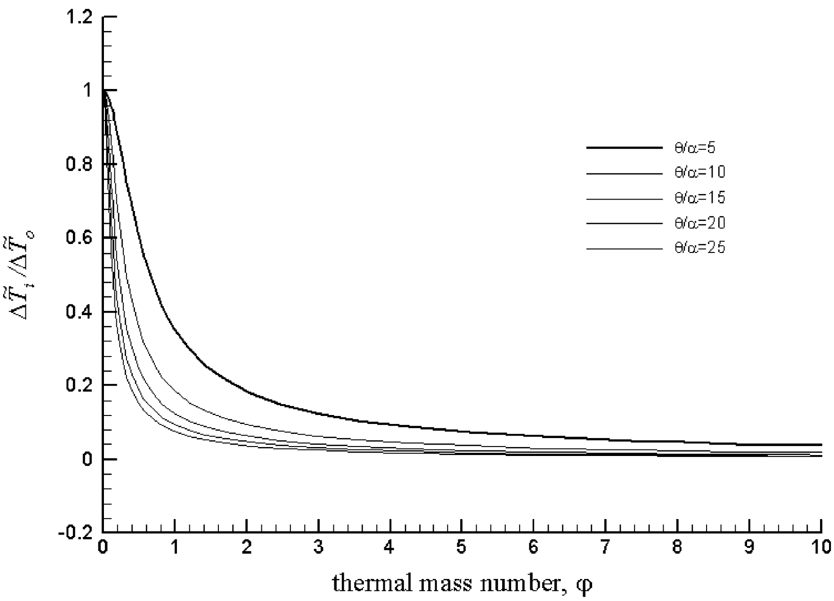

Fig. 11. The ratio of indoor air temperature fluctuation to outdoor temperature fluctuation as a function of the thermal mass number for the stack-ventilated model with $T_M = T_i$.

$$\frac{\varphi}{\sqrt[3]{2}}\frac{\theta}{\alpha}\frac{\partial Y}{\partial(\omega t)}+\frac{3}{2}\varphi\frac{\theta}{\theta_H}|Y|^{1/2}\frac{\partial Y}{\partial(\omega t)}+Y|Y|^{1/2}=1-\varphi\cos(\omega t) \tag{29}$$

where $Y=(T_i-T_o)/\theta_E$ and $\theta_H=h_MA_M/\rho C_p$, which is the convective heat transfer air change parameter. As θ_H goes to zero, there is no thermal link between the thermal mass and the air temperature. The indoor air temperature is then in phase with the outdoor temperature.

Eq. (29) can be rearranged as

$$\varphi\left(\frac{\theta}{\alpha}\right)\left[\frac{1}{\sqrt[3]{2}}\left(\frac{\theta_H}{\alpha}\right)+\frac{3}{2}|Y|^{1/2}\right]\frac{\partial Y}{\partial(\omega t)}+\left(\frac{\theta_H}{\alpha}\right)Y|Y|^{1/2} = \left(\frac{\theta_H}{\alpha}\right)[1-\varphi\cos(\omega t)] \tag{30}$$

We can also obtain from Eq. (14),

$$T_i=T_o+\frac{q|q|}{2gh(C_dA^*)^2}\widetilde{T}_o \tag{31}$$

Substitute Eq. (31) into (27), we obtain

$$T_M=\frac{1}{\theta_H}\frac{E}{\rho C_p}\left(\frac{q^3}{2\alpha^3}+\theta_H\frac{q|q|}{2\alpha^3}-1\right)+T_o \tag{32}$$

Substituting Eqs. (31) and (32) into Eq. (8), after rearrangement, we have

$$\varphi\left(\frac{\theta}{\alpha}\right)\left[\frac{3}{2}\left(\frac{q}{\alpha}\right)^2+\left(\frac{\theta_H}{\alpha}\right)\left|\frac{q}{\alpha}\right|\right]\frac{\partial(q/\alpha)}{\partial(\omega t)}+\frac{1}{2}\left(\frac{\theta_H}{\alpha}\right)\left(\frac{q}{\alpha}\right)^3 = \left(\frac{\theta_H}{\alpha}\right)[1-\varphi\cos(\omega t)] \tag{33}$$

The governing equation for the thermal mass temperature T_M is not explicitly given here as it is not of our primary interest and additional complexity arises due to the appearance of higher order terms if q and T_i are expressed in terms of T_M.

As the amount of thermal mass approaches infinity, both the thermal mass number, φ, and the convective heat transfer air change parameter, θ_H, approach infinity. Hence, Eq. (29) becomes

$$\frac{\theta}{\sqrt[3]{2}\alpha}\frac{\partial Y}{\partial(\omega t)}=-\cos(\omega t) \tag{34}$$

the solution will be

$$Y=C_1-\frac{\sqrt[3]{2}\alpha}{\theta}\sin(\omega t) \tag{35}$$

where C_1 is a constant. As $Y=\frac{T_i-T_o}{\theta_E}$, we obtain

$$T_i=T_o+C_1\theta_E-\frac{\sqrt[3]{2}\alpha}{\theta}\theta_E\sin(\omega t)\quad\text{or}\quad T_i=\widetilde{T}_o+C_1\theta_E \tag{36}$$

Thus the indoor air temperature becomes a constant as the thermal mass value approaches infinity.

At the same time, as the thermal mass approaches infinity, the airflow rate becomes

$$\left(\frac{\theta}{\alpha}\right)\left|\frac{q}{\alpha}\right|\frac{\partial(q/\alpha)}{\partial(\omega t)}=-\cos(\omega t)$$

We can obtain $q|q|=C_2-(2\alpha^3/\theta)\sin(\omega t)$, the time lag for the flow rate is 12 h.

A number of different parameters are emerged to describe the effect of thermal mass in buildings with ventilation. When the ventilation rate is constant, both the phase shift and fluctuation of the indoor temperature are determined by the time constant of the system $\tau = MC_M/\rho C_p q$ and the dimensionless convective heat transfer number, $\lambda = h_M A_M/\rho C_p q$. λ is analog to the Biot number. Recall that a small value of the Biot number means the external resistance (convective heat transfer) being large compared to the internal resistance (heat conduction) and in this case the internal temperature distribution can be assumed to be uniform. Similarly, a large value of the convective heat transfer number means that the convective heat transfer is very effective compared to the flow mixing in the room, and the thermal mass temperature can thus be considered as in equilibrium with the room air temperature.

When the ventilation rate is a function of indoor and outdoor air temperature difference, the characteristic parameters are quite different, we suggest the use of

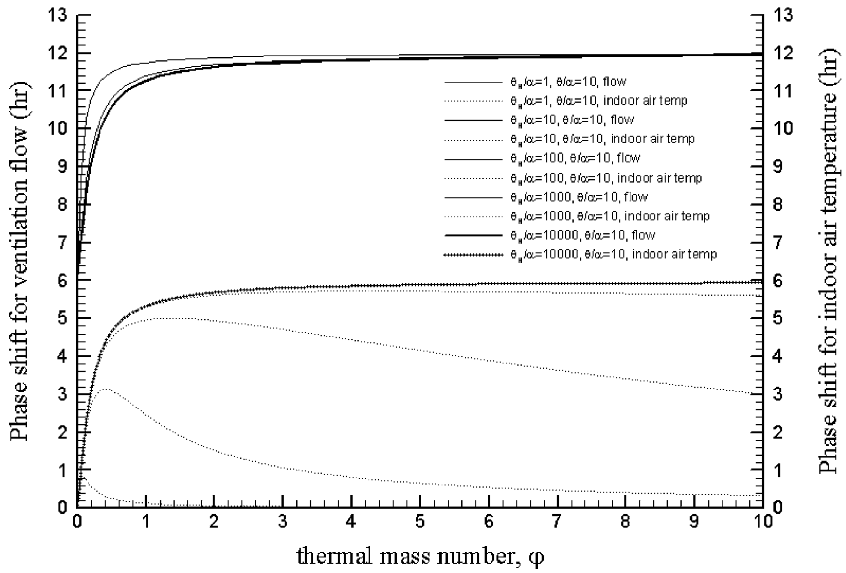

Fig. 12. Phase shifts for both the ventilation flow rate (solid lines) and the indoor air temperature (dotted lines) for the stack-ventilated model with $T_M \neq T_i$.

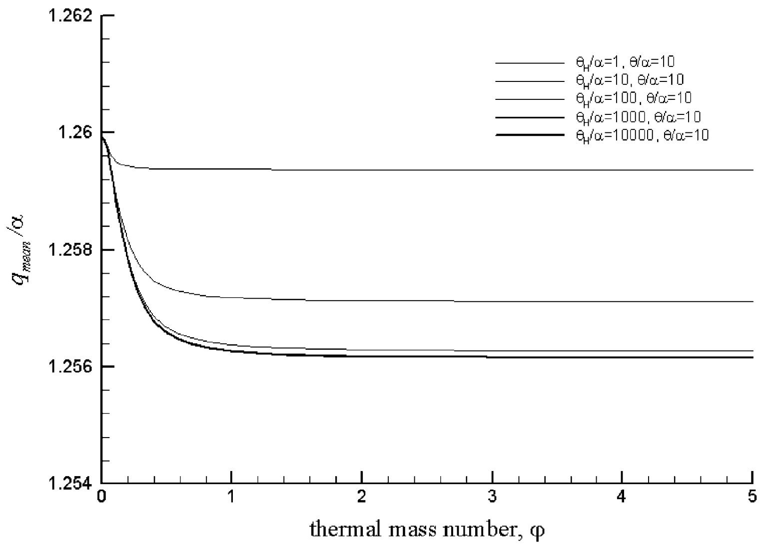

Fig. 13. Normalized mean ventilation rate as a function of the thermal mass number for the stack-ventilated model with $T_M \neq T_i$.

thermal mass number, φ and convective heat transfer air change parameter, θ_H. The thermal mass number measures the capacity of heat storage, rather than the amount of thermal mass. By calculating the value of these non-dimensional numbers or parameters, engineer can make an estimate of whether the thermal mass design will be suitable for its application.

Fig. 12 shows the changes of phase shift for both the airflow rate and the indoor air temperature for $\theta/\alpha = 10$, it was observed that the results for other values of θ/α are similar. As shown in Fig. 12, when $\theta_H > 1000$, the phase shift curves approach to that for the situation when the thermal mass is in equilibrium with the indoor air. However, a smaller value of the normalized con-

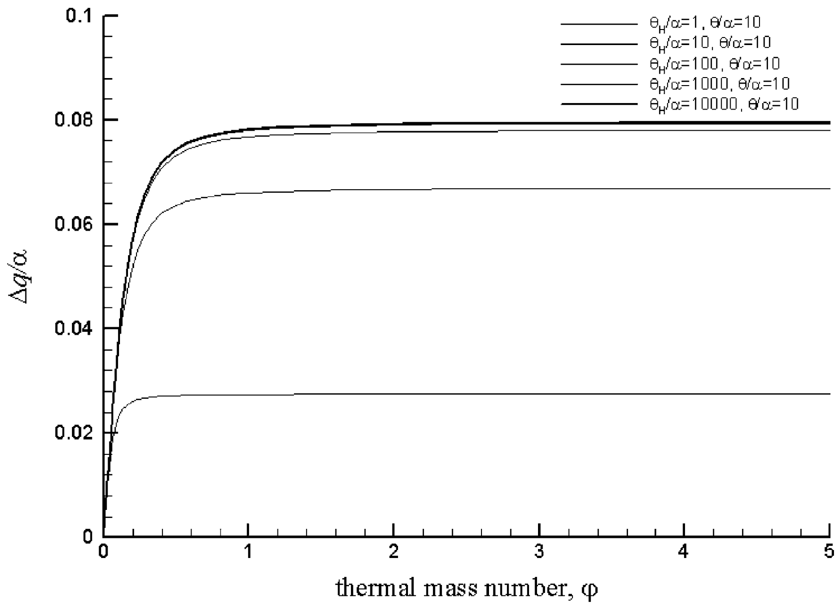

Fig. 14. Normalized ventilation rate fluctuation as a function of the thermal mass for the stack-ventilated model with $T_M \neq T_i$.

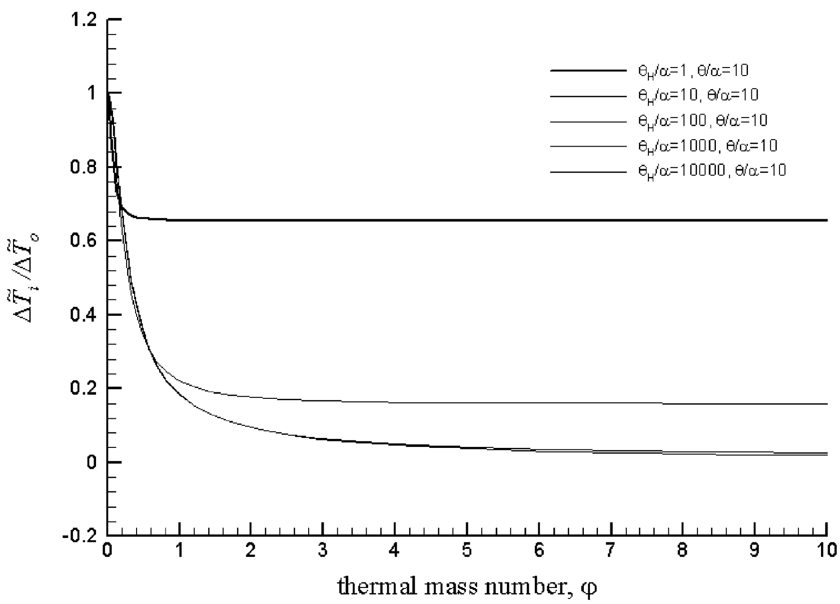

Fig. 15. The ratio of indoor air temperature fluctuation to outdoor temperature fluctuation as a function of the thermal mass number for the stack-ventilated model with $T_M \neq T_i$.

vective heat transfer air change parameter reduces the phase shift significantly. The results are very similar to those shown in Fig. 5 for the phase shift of indoor air temperature.

Figs. 13 and 14 show the normalized mean flow rate and flow fluctuation (amplitude) in terms of the thermal mass number respectively. The effect of the convective heat transfer parameter θ_H/α is also shown. Similar to the results as shown in Figs. 9 and 10, the ventilation flow rate equals $\sqrt[3]{2}\alpha$ and there is no fluctuation when the effect of thermal mass is absent (i.e. $\varphi = 0$). These results can be shown as well by examining the governing equations (27) and (8). As the thermal mass number, φ, increases beyond the value of 1, the predicted mean flow rate and flow fluctuation depends only on θ_H for constant θ/α ratio. However, the results for large values of θ_H/α (greater than 100) are approximately the same.

In Fig. 15, the normalized indoor air temperature fluctuation is shown as a function of φ for different values of the normalized convective heat transfer air change parameter θ_H/α. When θ_H/α is small (less than 10), the normalized indoor air temperature fluctuation is controlled by the convective heat transfer process at the thermal mass surfaces. Further increase of thermal mass has no effect at all. Again, this suggests that the importance of the convective heat transfer between the mass and the indoor air.

4. Conclusion

It is possible to design a system with a predicted time lag for a naturally ventilated building using the simple methods derived in this paper.

Unlike the periodic heat transfer through the building envelope which can introduce a large time lag for the indoor air temperature, we have shown here, the maximum indoor air temperature phase shift induced by the direct outdoor air supply *without control* is 6 h in an either mechanically or naturally ventilated building. The non-linearity of the system does not significantly change the periodic behaviour of the system and the phase shift of the indoor air temperature when a periodic outdoor air temperature profile is considered. When the ventilation rate is constant, both the phase shift and fluctuation of the indoor temperature in a simple building are determined by the time constant of the system and the dimensionless convective heat transfer number. When the ventilation rate is a function of indoor and outdoor air temperature difference, the so-called "thermal mass number" and "convective heat transfer air change parameter" are suggested.

Appropriate amount of thermal mass should be used in building passive design as further increase of thermal mass would not increase the phase shift of the system. Convective heat transfer at the thermal mass surfaces is very important and its quantitative impact shows that the phase shift of the indoor air temperature can be reduced even the thermal mass number is increased if the convective heat transfer number is less than 10.

It is hoped that the present study will pave the way for further understanding of the thermal mass effect and develop design guidelines for natural ventilation and passive design of buildings.

Acknowledgements

This project is a contribution to the International Energy Agency Annex 35 project (Hybvent). The research was supported by a grant of the Hong Kong Research Grants Council (HKU 7009/01E). The authors would like to thank Dr. Angelo Delsante for his valuable discussions on the problem. Dr. Delsante first derived the solutions when the thermal mass number approaches to infinity while he visited the University of Hong Kong in November 2001.

Appendix A. Mathematical analyses for the non-linear governing equation

The non-linear governing equation for the ventilation flow rate derived in Section 3.1 is as follows:

$$2\varphi\theta|q|\frac{\partial q}{\partial(\omega t)} = -q^3 + 2\alpha^3[1 - \varphi\cos(\omega t)] \tag{A.1}$$

where $\varphi > 0$, $\theta > 0$ and $\alpha > 0$. The non-linearity of the equation arises as a result of the coupling between indoor air temperature and ventilation flow rate. The above equation was derived assuming that the outdoor air temperature changes sinusoidally. The question is whether the airflow rate and indoor air temperature also change sinusoidally and what will be the phase shift from the outdoor air temperature for both ventilation flow rate and the indoor air temperature.

Unfortunately, Eq. (A.1) cannot be solved analytically and we have carried out a mathematical analysis using some concepts from the dynamic systems theory. The detailed mathematical proof is not given here, as it is too lengthy. The results are summarized as follows.

Eq. (A.1) can be written as

$$\begin{cases} \dfrac{ds}{d\tau} = 2\varphi\theta|q| \\ \dfrac{dq}{d\tau} = -q^3 + 2\alpha^3[1 - \varphi\cos s] \end{cases} \tag{A.2}$$

where $s = \omega t$. Let $(s(\tau; s_0, q_0), q(\tau; s_0, q_0))$ represents the solution of the system (A.2) when the initial condition is (s_0, q_0) at $\tau = 0$. Let $\gamma(s_0, q_0)$ represents the whole trajectory of the system (A.2) that passes through (s_0, q_0) at $\tau = 0$. Let $\gamma^+(s_0, q_0)$ and $\gamma^-(s_0, q_0)$ represent respectively the positive and negative half trajectory of the system (A.2) that pass through (s_0, q_0) when $\tau = 0$. Let $\frac{ds}{d\tau}|_{(s,q)}$ and $\frac{dq}{d\tau}|_{(s,q)}$ represent the values of $ds/d\tau$ and $dq/d\tau$ at (s, q) respectively. Furthermore, we let

$$H = \Big\{(s,q)| - \infty < s < +\infty, \sqrt[3]{2\alpha^3(1-\varphi)} \leqslant q \leqslant \sqrt[3]{2\alpha^3(1+\varphi)}\Big\} \quad \text{(A.3)}$$

Our analyses are divided into three steps for three different situations, $0 < \varphi < 1$, $\varphi = 1$ and $\varphi > 1$.

Step 1: To show that for any initial conditions, the transient solution will fluctuate within a range of parameters. Mathematically, this can be written as

Theorem 1. $\forall (s_0, q_0) \in R^2$, $\exists T_o > 0$ *such that*

$$\{(s(\tau; s_0, q_0), q(\tau; s_0, q_0))|\tau \geqslant T_o\} \subset H \quad \text{(A.4)}$$

and when $\tau \geqslant T_o$, $(s(\tau; s_0, q_0), q(\tau; s_0, q_0))$ *oscillates within* H.

Step 2: To show that the transient solution is a periodic solution with a period of 24 h. Again, mathematically, we can write the following statement.

Theorem 2. *The solution for Eq.* (A.2) *is a periodic oscillating solution* γ^* (*q oscillates periodically with respect to s*) *for the differential equations set* (A.2) *with a* 24-*h period*, $\gamma^* \subset H$, *and* γ^* *is a global attractor*.

Step 3: To determine the phase shift between the periodic solution of ventilation flow rate and the outdoor air temperature.

Theorem 3. *The phase difference between the periodic oscillating solution and* T_o *satisfies* 6 *h* $< \Delta\phi <$ 12 *h*.

The above three theorems show that the solution for the ventilation flow rate is periodic with a period of 24 h and a phase shift between 6 and 12 h and they apply to the situation when $0 < \varphi < 1$. Similar but more complex conclusions can be obtained when $\varphi = 1$ and $\varphi > 1$.

When $\varphi = 1$, there is also one dynamic solution which approaches to the static solution which is not stable, and all other dynamic solutions approach to γ^*. When $\varphi = 1$, the phase difference between the periodic oscillating solution γ^* and T_0 also satisfies 6 h $< \Delta\phi <$ 12 h.

When $\varphi > 1$, equation set (A.2) at least satisfies one of the following two results:

(1) there exists a periodic oscillating solution γ^* (*q* oscillates periodically with *s*), with a 24-h period (with respect to *t*), $\gamma^* \subset H$, and the phase difference between γ^* and T_o satisfies 6 h $< \Delta\phi <$ 12 h, or
(2) within every period there exists a trajectory, with its negative side approaches to the static solution $(2k\pi + \cos^{-1}(\frac{1}{\varphi}), 0)$, and positive side approaches to the static solution $(2(k+1)\pi + \cos^{-1}(\frac{1}{\varphi}), 0)$. This trajectory together with the static solution make up the whole periodic oscillating curve η, with a period of 24 h (with respect to *t*), $\eta \subset H$, and the phase difference between η and T_o satisfies 6 h $< \Delta\phi <$ 12 h.

Similar analyses can be done to show that the solution for the indoor air temperature is also periodic with a period of 24 h and a phase shift between 0 and 6 h. These analyses are useful as it is impossible to solve analytically the non-linear governing equations for all range of influencing parameters. It should be noted that we failed to prove that the solution is unique for all situations, although it seems that the numerical periodic solution is indeed unique as shown in the main text.

References

[1] C.A. Balaras, The role of thermal mass on the cooling load of buildings. An overview of computational methods, Energy Build. 24 (1) (1996) 1–10.
[2] P. Burberry, Practical Thermal Design in Buildings, Batsford Academic and Educational Ltd, London, 1983.
[3] E.H. Mathews, P.G. Rousseau, P.G. Richards, C. Lombard, A procedure to estimate the effective heat storage capability of a building, Build. Environ. 26 (2) (1991) 179–188.
[4] B. Givoni, Man, Climate and Architecture, second ed., Applied Science, London, 1981.
[5] H.S. Carslaw, J.C. Jaeger, Conduction of heat in solids, second ed., Clarendon Press, Oxford, 1959.
[6] E. Danter, Periodic heat flow characteristics of simple walls and roofs, JIHVE 28 (1960) 136–146.
[7] J. Maloney, T. Wang, B. Chen, J. Thorp, Thermal network predictions of the daily temperature fluctuations in a direct gain room, Solar Energy 29 (3) (1982) 207–223.
[8] H. Asan, Y.S. Sancaktar, Effects of wall's thermophysical properties on time lag and decrement factor, Energy Build. 28 (2) (1998) 159–166.
[9] J.L. Threlkeld, Thermal Environmental Engineering, second ed., Prentice-Hall, New Jersey, 1970.
[10] J. van der Maas, C.-A. Roulet, Nighttime ventilation by stack effect, ASHRAE Trans. 97 (1) (1991) 516–524.
[11] P. Blondeau, M. Spérandio, F. Allard, Night ventilation for building cooling in summer, Solar Energy 61 (5) (1997) 327–335.
[12] R. Kammerud, E. Ceballos, B. Curtis, W. Place, B. Andersson, Ventilation cooling of residential buildings, ASHRAE Trans. 90 (1B) (1984) 226–252.
[13] E.H. Mathews, Thermal analysis of naturally ventilated buildings, Build. Environ. 21 (1) (1986) 35–39.
[14] G.E. Myers, Analytical Methods in Conduction Heat Transfer, Genium Publishing Corporation, Schenectady, New York, 1987.
[15] Y. Li, A. Delsante, Natural ventilation induced by combined wind and thermal forces, Build. Environ. 36 (1) (2001) 59–71.